MEC 542: Mechanics of Deformable Bodies

1.0 Introduction

The topic "Mechanics of deformable bodies" deals with the study of resulting stresses and strains developed in bodies as a result of external influences or actions. This study provides us with answers to questions like:

(i) How much deformation will take place in/on a body when subjected to a certain external force.
(ii) How much force can a body withstand without failing?
(iii)What is the best shape a body should take to withstand failure?

Engineers and other scientists, apply the knowledge of stress analysis in the design of systems such as automobiles, missiles, aircrafts, ships, submarines, machines, etc. Basically, any matter whose strength, weight or deformation of important consideration is studied from the point of view of stress analysis. In theory, this course describes the use differential equation and solutions for the investigation of matters of different shapes and materials, under different conditions of stress and strain, caused by the action of various external forces.

1.1 Basic Definitions

1.1.0 Force: force is defined as a push or pull. In stress analysis, forces are classed into two kinds. They are:- the surface forces and the body forces.

Surface Forces: This forces result as a result of the action of a body's surface on the other when in contact.

Body Forces: this is a type of force that acts on the volume of each mass and it is independent of the surrounding material or environment.

Figure 1.0: Resultant force and the components of force.

1.1.1 Stress

The intensity/effect of force on a body is dependent on the area of the surface it acts on. This intensity is called traction and it is normally represented in terms of its components, parallel and perpendicular to a surface.

Figure 1.1a: Traction component

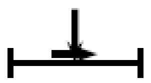

Figure 1.1b: Traction components

Since mechanical requirement equilibrium must be satisfied, the surface must have equal and opposite tractions acting on the opposite surfaces. This pair of traction defines the surface stress vector Σ or **S** which is defined by the total force exerted on the surface of a body divided by the surface area of the body A.

$$\Sigma = S = \frac{\Sigma F}{A} \tag{1.1}$$

Consider a situation where the resultant force acts on an area in such a way that A→0, the stress vector on the surface area exposed to the resultant force will be defined as:

$$\text{Stress Vector} = \Sigma = \lim A \rightarrow 0 \frac{\Sigma F}{A} \tag{1.2}$$

Stress and traction are measured in N/m^2, Pa or bar.

The vector stress Σ is made up of the normal **(σ)** and tangential **(τ)** stress components and can be thus resolved.

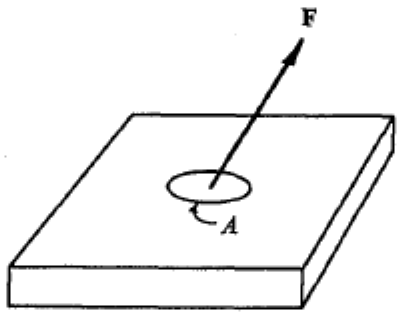

Figure 1.2a: Normal stress component (**σ**)

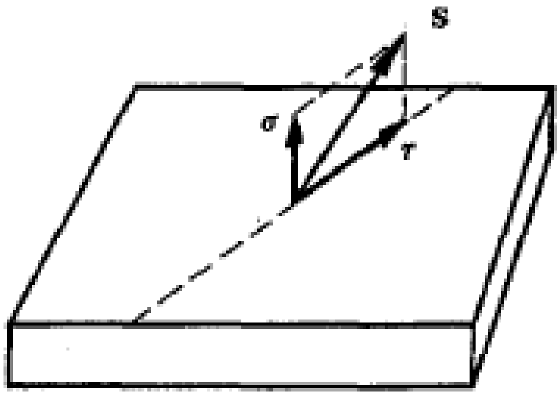

Figure 1.2b: Tangential stress component (**τ**)

1.1.2 Strain

Strain defines the change in dimensions and shapes of materials. The basic definition of strain is when the deformation takes place along one axis. It is expressed thus:

$$Strain = \frac{Change\_in\_Length}{Original\_Length} \quad \textbf{(1.3)}$$

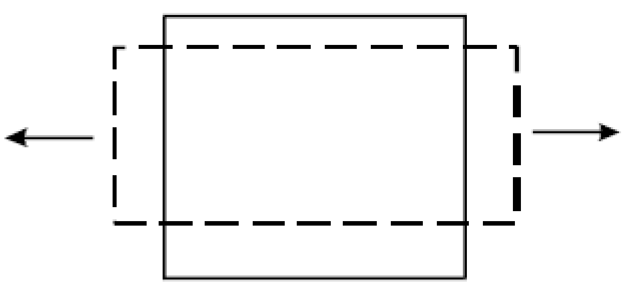

Figure 1.3a: Normal Strain

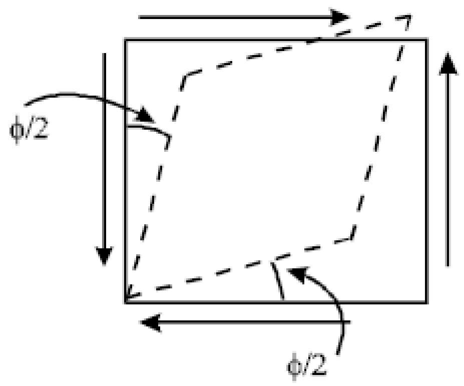

Figure 1.3b: Shear Strain

When a material is stretched, the strain is positive and when it is compressed, the strain is negative. This supports the signs of the stresses which would accompany these strains, tensile stresses being positive and compressive stresses negative. A material is said to undergone normal strain if there are changes in the materials dimension but not the shape, this implies that the material's angles do not change. In general, normal strains occur along the three mutually perpendicular axes.

1.2 Constitutive Equations

A body under the influence of external forces is a continuum and the material's behaviour is mathematically characterized by **constitutive equations** also known as **material laws**. The following assumptions are made on material behaviours:

1.2.0 Material Behaviour Assumptions

1.2.1 Macroscopic Model: All materials under examination are mathematically modelled as a continuum body. Nano, micro and meso level materials such as atoms, molecules and grains are neglected.

1.2.2 Elasticity: this implies that stress-strain response is reversible and consequently, the material has a preferred natural state. The natural state is referred to a state of the material at a reference temperature and with no load acting on it.

1.2.3 Linearity: the relationship between strain and stress for all materials is linear. (Hooke's law).

1.2.4 Small Strains: Changes in the geometry of a body is neglected when deformation is considered to be very small. This assumption does not apply to highly deformable materials like rubber and some polymers.

1.2.5 Isotropy: The properties of the material are independent of direction. This assumption is suitable for materials such as metals, concrete, plastics, etc. For heterogeneous mixtures such as composites or reinforced concrete, which are *anisotropic* by nature is not suitable.

1.3 Tension Test

Ideally, when a tension test is carried out on mild steel, the stress-strain behaviour varies with increase in load to the point of failure. The behaviour of the stress-strain relationship or the materials responses to loads are classed into regions: the elastic (linear), yield, strain hardening, localization and failure regions. In this elastic region, the one-dimensional Hooke's law is assumed to hold.

Some engineering materials deviate from the above mentioned behaviour in response to loading. Some of the materials that fall into these categories are: - (i) the brittle (ii) moderately ductile and (iii) the non-linear from start. Even though such materials could be elastic, their linear elastic region cannot be easily identified.

For a well known material like steel, the tension behaviour can vary significantly, depending on its composition. Mild steel is highly ductile and clearly exhibits an extensive yield region while Hi-strength steel does not show a will defined yield point because it is less ductile.

It is worthy to note that all the grades of steel have approximately the same elastic modulus, which is the slope of stress-strain line in the region of the tension test.

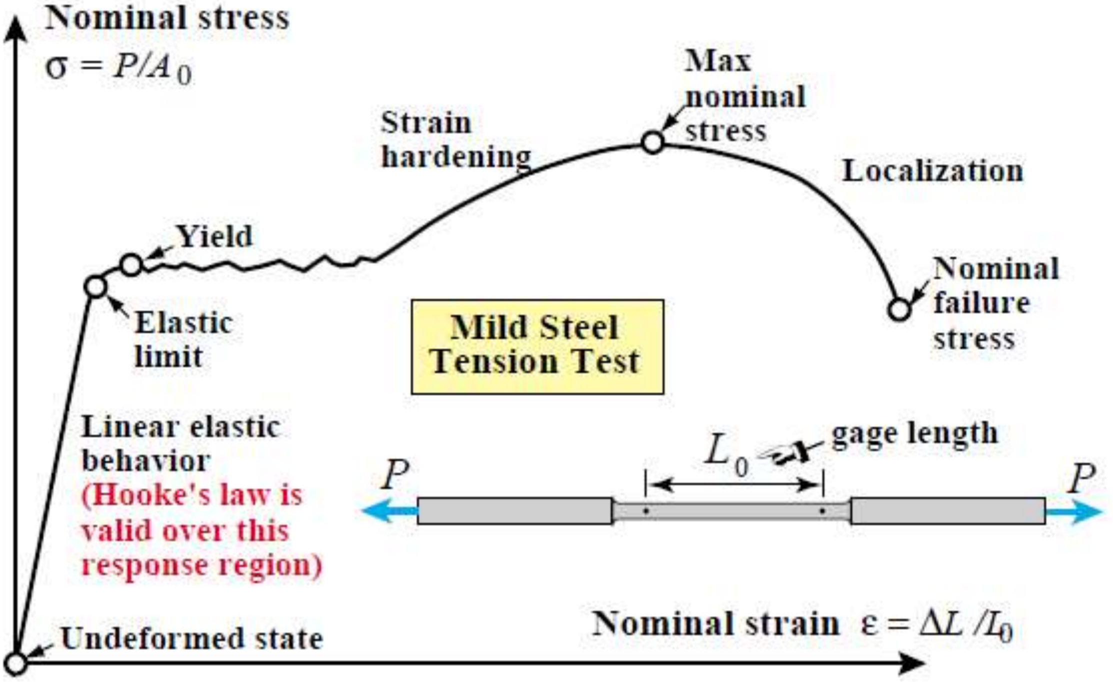

Figure 1.4: A typical tension test behaviour of mild steel with well defined yield point and extensive yield region.

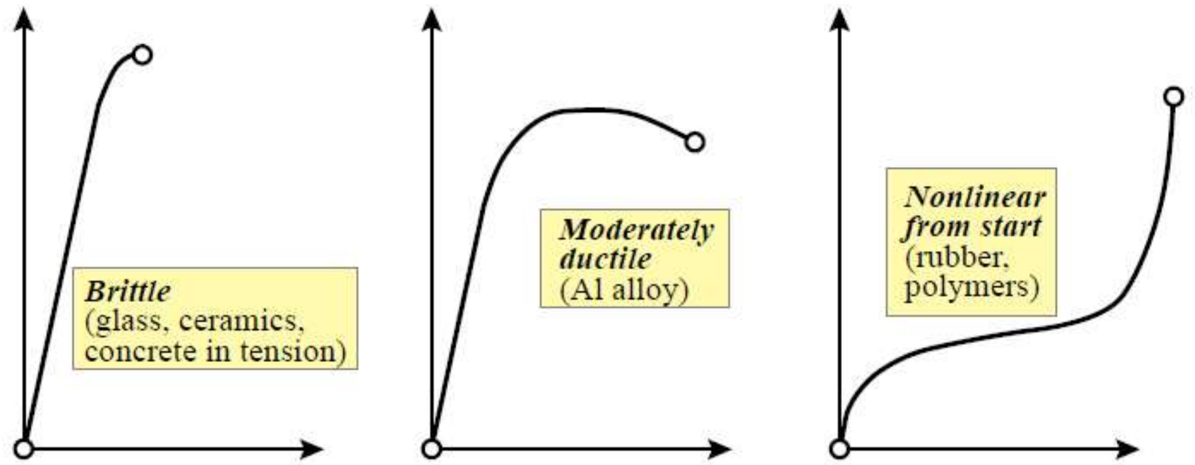

Figure 1.5: Three material responses during tension test

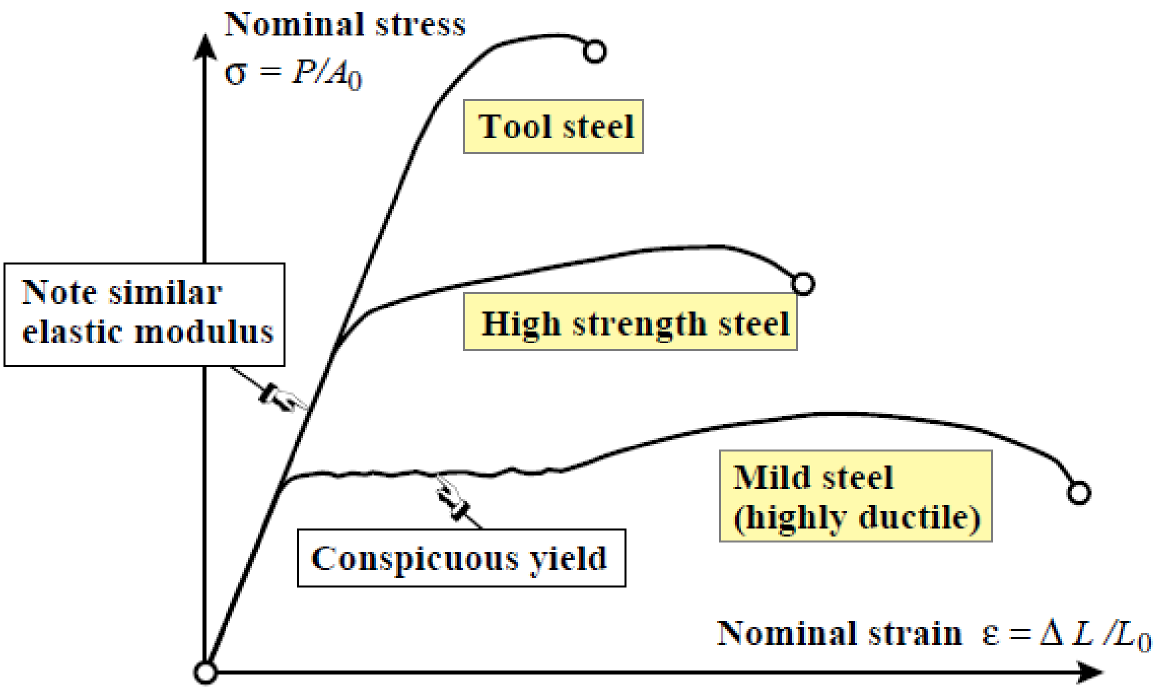

Figure 1.6: Response of three grades of steel to loads